Oscillating inflation with a nonminimally coupled scalar field

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The oscillating inflation model recently proposed by Damour and Mukhanov is investigated with nonminimal coupling. Numerical study confirms an inflationary behavior and density perturbation is obtained. Successful inflation requires the gravity-dilaton coupling to be small.

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Recently, Damour and Mukhanov [1] proposed an inflation model with oscillating inflaton fields in the Einstein gravity context. Meanwhile, dilatonlike fields are natural candidates for inflaton fields, because generally inflatons are supposed to be gauge singlets. Furthermore, the extended gravity sector is common to unified theories such as supergravity, superstring or M-theory, and Kaluza-Klein theory [2]. These dilaton fields might be stabilized by some potential, which could be an inflaton potential. In this respect, in this paper, we study oscillating inflation with nonminimal coupling.

We consider an action of nonminimally coupled scalar theory which is given by [3]

$$S=\int \sqrt{-g}d^4x\left[-U(\phi)R+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right], \quad (1)$$

where ϕ is a dilatonlike inflaton field and $U(\phi)=M_P^2/16\pi+\xi\phi^2/2$. We adopt the inflaton potential

$$V(\phi)=\frac{A}{q}\left[\left(\frac{\phi^2}{\phi_0^2}+1\right)^{q/2}-1\right], \quad (2)$$

which was suggested by Damour and Mukhanov [1]. Here A, q, and ϕ_0 are constants. The equations of motion for the scale factor a and ϕ from the action [Eq. (1)] with the metric $ds^2=-dt^2+a(t)^2dx^2$ are

$$3H^2\left[\frac{M_P^2}{8\pi}+\xi\phi^2\right]=\frac{\phi'^2}{2}+V(\phi)-6\xi H\phi\phi' \quad (3)$$

and

$$\phi''+3H\phi'-6\xi(H'+2H^2)\phi+\frac{dV}{d\phi}=0, \quad (4)$$

where the prime denotes d/dt and $H\equiv a'/a$. To see whether oscillating inflation really happens in this model, we do a numerical study. A typical signal of oscillating inflation is an increasing and wiggly curve of aH versus t. The result of the numerical calculation shows this curve (Fig. 1) and confirms inflationary behaviors.

Now let us show that $\xi<q\ll 1$ is a good parameter range for our model with the Damour-Mukhanov potential and calculate the density perturbation. The Cosmic Background Explorer (COBE) observation requires $\delta_H\simeq 2\times 10^{-5}$. Since the oscillating inflation phase is generally short, the observed

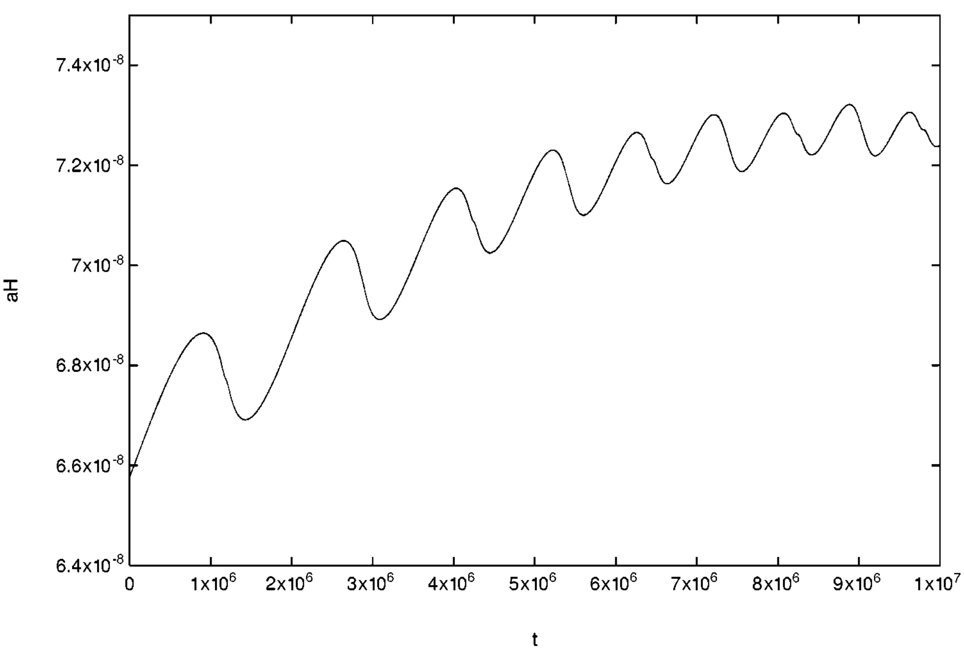

FIG. 1. aH vs t from numerical solutions of the field equations with $\xi=0.001$, $q=0.1$, $\phi_0=0.01$, and $A=10^{-14}$. All the quantities are denoted in natural units where $M_P=1$.

perturbation is supposed to be generated during a slow-roll phase. From Eqs. (3) and (4) and with slow-roll conditions one can obtain [4]

$$3H\phi' \simeq \frac{1}{1+\xi\kappa^2\phi^2(1+6\xi)} \times\left[4\xi\kappa^2\phi V(\phi)-(1+\xi\kappa^2\phi^2)\frac{dV}{d\phi}\right], \tag{5}$$

where $\kappa^2=8\pi/M_P^2$. Since during the slow-roll phase $\phi \gg \phi_0$, one can approximate the Damour-Mukhanov potential as

$$V(\phi)\simeq\frac{A}{q}\left(\frac{\phi}{\phi_0}\right)^q. \tag{6}$$

However, for $q \ll 1$ which is required for oscillating inflation [1], such a polynomial potential with a small exponent brings the following problem with nonminimal coupling [5]: When $q \ll 1$ the first term in Eq. (5) is larger than the second one and the former gives a force which prevents ϕ rolling down to the potential minimum. One can overcome this problem by simply choosing parameters satisfying

$$\xi\kappa^2\phi^2 < q/(4-q) \ll 1. \tag{7}$$

Since for the potential the number of e-folds of expansion during the slow roll is given by

$$N \simeq -\frac{8\pi}{M_P^2}\int\frac{V}{dV/d\phi}d\phi \simeq \frac{4\pi\phi^2}{qM_P^2}, \tag{8}$$

one can obtain a sufficient expansion with initial $\phi=\phi_N \sim M_P$. Then the above condition [Eq. (7)] becomes $\xi < 1/8N$. Furthermore, it is known that for $\xi \ll 1$, the density perturbation is proportional to $H^2/|\phi'|$ as usual [6]. Therefore, in this limit [7],

$$\delta_H^2 \simeq \frac{512\pi}{75}\frac{A}{q^3M_P^6}\frac{\phi^{2+q}}{\phi_0^q}\left[1+\xi\left(3q-\frac{3\kappa^2\phi^2}{2}+\frac{\kappa^2\phi^2}{q}\right)\right]^2 \tag{9}$$

is a good approximation up to $O(\xi)$ within the mentioned parameter ranges ($\xi < q \ll 1$). The term with ξ represents the effect of non-minimal coupling which is dominated by the last term $\xi\kappa^2\phi^2/q$ for $q \ll 1$. Therefore in the marginal case ($\xi\kappa^2\phi^2/q \simeq \frac{1}{4}$) this could contribute about $\frac{1}{4}$ to the factor of δ_H. Let us approximate δ_H^2 as

$$\delta_H^2(\xi\neq 0) \simeq \delta_H^2(\xi=0)\left[1+\frac{2\xi\kappa^2\phi^2}{q}\right]. \tag{10}$$

Since one can easily change the magnitude of δ_H by choosing constants in the potential such as A and ϕ_0, one needs to observe the spectral index n of the perturbation rather than the magnitude of it. The background radiation observations MAP [8] and Planck [9] will be a discriminator between many inflation models. Since

$$n=1+\frac{25}{4}\frac{d\delta_H^2}{d\ln k}, \tag{11}$$

where k is a wave number of the perturbation [10], the deviation of n between the oscillating inflation with Einstein gravity and with nonminimal coupling is

$$\Delta n = \frac{25}{4}\frac{d[\delta_H^2(\xi\neq 0)-\delta_H^2(\xi=0)]}{d\ln k} \simeq \frac{25}{2}\frac{d\phi}{d\ln k}\frac{d(\xi\kappa^2\phi^2/q)}{d\phi}\delta_H^2(\xi=0) + \frac{2\xi\kappa^2\phi^2}{q}[n(\xi=0)-1], \tag{12}$$

where $d\phi/d\ln k = -(M_P^2/8\pi V)dV/d\phi$. The first term contributes $-10^{-10}\xi$ to Δn which is too small to be observed by satellites. Using $1-n \simeq (q+2)/2N$ for chaotic inflation with $V\sim\phi^q$ [11], one can find that even in the marginal case the second term is about $-\xi\kappa^2\phi^2/30q \simeq 1/120$, which is rather smaller than the accuracy of Planck, $\Delta n \sim 0.02$. So it is hard to expect that the observations could distinguish between the two models the near future.

Now let us check the inflation conditions during the oscillating phase. The scale factor also satisfies the following equation:

$$\frac{a''}{a}=\frac{\kappa^2}{1+\xi\kappa^2\phi^2}[-\phi'^2(1+3\xi)+V(\phi)-3\xi\phi(H\phi'+\phi'')]. \tag{13}$$

We adopt the reasonable assumption that during the oscillation the friction term is negligible ($\phi'' \gg 3H\phi'$) and the effective mass squared of the field [see Eq. (4)]

$$m^2 \equiv \frac{d^2V(\phi)}{d\phi^2}-6\xi(H'+2H^2) \gg H^2. \tag{14}$$

Since the typical period of an oscillation is $1/m$, one can ignore the variation of H (and the H'-dependent term) during a single oscillation. One can also easily find that in Eq. (14), the ξH^2-dependent term should be small relative to $d^2V/d\phi^2$, especially when $\xi \ll 1$. So it is a good approximation that $\phi'' \simeq -dV/d\phi$. Since we expect that the present value of $U(\phi)$ is equal to $M_P/16\pi$, the present value of ϕ should be 0. So the limit $\xi\kappa^2\phi^2 \to 0$ is a reasonable assumption near the potential minimum and for $\xi \ll 1$. In this limit and with $\langle\phi\phi''\rangle = \langle -\phi'^2\rangle$ the inflation condition is

$$\left\langle\frac{a''}{a}\right\rangle \simeq \langle\kappa^2[-\phi'^2(1+3\xi)+V(\phi)-3\xi\phi(H\phi'+\phi'')]\rangle \simeq \kappa^2\langle -\phi'^2+V(\phi)\rangle > 0, \tag{15}$$

where the angular brackets denote the time average during an oscillation period.

With $\phi''=-dV/d\phi$ this reduces to

$$\left\langle V-\phi\frac{dV}{d\phi}\right\rangle>0, \tag{16}$$

which is the result of Ref. [1]. Of course, this is simply due to the fact that in this limit the system becomes that with Einstein gravity. So our model is self-consistent at least when $\xi<q\ll 1$.

In summary, in the context of nonminimal coupling, we investigated the oscillating inflation and calculated the density perturbation during the slow-roll phase with the Damour-Mukhanov potential.

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